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PLATO'S MATHEMATICAL CONSTRUCTION

I. THE PUZZLE

A relatively well-known puzzle in the history of Greek mathematics arises from Eutocius' ascription to Plato, in the context of a commentary on Archimedes, of a mechanical solution to the problem of finding two mean proportionals.[1] That a mathematical text by Plato may have existed in antiquity, have survived until the sixth century A.D., and yet have left no other trace before or since, is incredible. On the other hand, there is no doubt that Eutocius intended his ascription—and so, probably, did his source—to refer to the famous philosopher Plato, and not to some otherwise unknown flat-footed ancient author. Other things being equal, a Plato is a Plato is a Plato.[2] Thus Eutocius certainly makes a false ascription, most probably reflecting one in a lost source. The arising puzzle is twofold: first, to identify the real author of the solution; second, to explain how the solution could have been falsely ascribed to Plato. The first part of the puzzle was discussed by Knorr (who, for reasons that will become apparent below, considers the solution to be ultimately by Eudoxus).[3] In this article, I concentrate on the second part of the puzzle.

That something meaningful can be said concerning such questions of ascription—always very difficult to analyse—is due to the presence of another puzzle in the immediate vicinity of this one. Putting the two puzzles together, some pattern may emerge, possibly offering clues for the solution.

II. THE EVIDENCE[4]

The text translated here appears at the beginning of Eutocius' catalogue of solutions to the problem of finding two mean proportionals (commenting upon a passage where Archimedes assumes the problem is solved). Before proceeding with his list of solutions, Eutocius offers a brief introduction, where our further clues may be found. I now translate this introduction, followed by 'Plato's' solution.

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This being taken, now that he has advanced through analysis the <terms> of the problem—the analysis terminating <by stating> that it is required, given two <lines>, to find two mean proportionals in continuous proportion—he says in the synthesis: 'let them be found', the finding of which, however, we have not found at all proved by him, but we have come across writings by many famous men that offered this very problem (of which, we have refused to

[1] J. L. Heiberg, *Archimedis opera omnia* III (Teubner, 1915), 56.13–58.14. The text, as well as its context, is translated below.

[2] To clarify the point: a well-known example in the theory of pragmatics discusses a placard carried next to the White House, in the early 1970s, with the words 'Nixon is an idiot'. It would be perverse, the example suggests, to take the placard to refer to some Nixon other than the then occupant of the White House: the pragmatic co-ordinates are too powerful. The same holds for the name 'Plato' in an ancient philosophical/scientific context.

[3] W. R. Knorr, *The Ancient Tradition of Geometric Problems* (New York, 1986), 57–61; id., *Textual Studies in Ancient and Medieval Geometry* (Boston, 1989), 78–80.

[4] The following translation of Heiberg (n. 1), 54.26–58.14 follows the conventions used, for example, in R. Netz, 'Archimedes transformed: the case of a result stating a maximum for a cubic equation', *Archive for History of Exact Sciences* 54 (1999), 1–47. In particular, I mark, for ease of reference, steps of the construction by Latin letters, and steps of the proof by Arabic numerals.

accept the writing of Eudoxus of Cnidus, since he says in the introduction that he has found it through curved lines, while in the proof, in addition to not using curved lines, he finds a discrete proportion and uses it as if it were continuous, which is absurd to conceive, I do not say for Eudoxus, but for those who are even moderately engaged in geometry). Anyway, so that the thought of those men who have reached us will become well known, the method of finding of each of them will be written here, too.

As Plato

Given two lines, to find two mean proportionals in continuous proportion.

Let the two given lines, whose two mean proportionals it is required to find, be $AB\Gamma$, at right <angles> to each other. (a) Let them be produced along a line towards Δ, E,[5] (b) and let a right angle be constructed,[6] the <angle contained> by $ZH\Theta$, (c) and in one side, e.g. ZH, let a ruler, $K\Lambda$, be moved, being in some groove in ZH, in such a way that it shall, itself <=$K\Lambda$>, remain throughout parallel to $H\Theta$. (d) And this will be, if another small ruler be imagined, too, fitting with ΘH, parallel to ZH: e.g. ΘM; (e) for, the upward surfaces[7] of ZH, ΘM being grooved in axe-shaped grooves (f) and knobs being made, fitting $K\Lambda$ to the said grooves, (1) the movement of the <knobs>[8] $K\Lambda$ shall always be parallel to $H\Theta$. (g) Now, these being constructed, let one chance side of the angle be set out, $H\Theta$, touching the <point> Γ,[9] (h) and let the angle and the ruler $K\Lambda$ be moved to such a position where the point H shall be on the line $B\Delta$, the side $H\Theta$ touching the <point> Γ,[10] (i) while the ruler $K\Lambda$ should touch the line BE on the <point> K, and on the remaining side[11] <it should touch> the <point> A,[12] (j) so that it shall be, as in the diagram: the right angle <=of the machine, namely ΘHK> has <its> position as the <angle contained> by $\Gamma\Delta E$, (k) and the ruler $K\Lambda$ has <its> position as EA has;[13] (2) for, these being made, the <task> set forth will be <done>. (3) For the <angles> at Δ, E being right, (4) as ΓB to $B\Delta$, ΔB to BE and EB to BA.[14]

[5] For the time being, Δ, E are understood to be as 'distant as we like'. Later the same points come to have more specific determination.

[6] The word—*kataskeuasthō*—is not part of normal geometrical discourse, and already foreshadows the mechanical nature of the following discussion. Notice also that we have now transferred to a new figure.

[7] 'Upward surfaces': notice that the contraption is seen from above (otherwise, of course, there is nothing to hold $K\Lambda$ from falling).

[8] The manuscripts—not Heiberg's edition—have a plural article, which I interpret as referring to the knobs.

[9] Imagine that what we do is to put the contraption on a page containing the geometrical diagram. So we are asked to put the machine in such a way that the side $K\Lambda$ touches the point Γ. This leaves much room for manoeuvre; soon we will fix the position in greater detail.

[10] The freedom for positioning the machine has been greatly reduced: H, one of the points of $H\Theta$, must be on the line $B\Delta$, while some other point of $H\Theta$ must pass through Γ. This leaves a one-dimensional freedom only: once we decide on the point on $B\Delta$ where $H\Theta$ stands, the position of the machine is given. Each choice defined a different angle $\Gamma\Delta B$. (Notice also that it is taken for granted that $H\Theta$ is not shorter than $B\Gamma$.)

[11] 'The remaining side' means somewhere on the ruler $K\Lambda$, away from K and towards Λ, though not necessarily at the point Λ itself.

[12] The point K must be on BE, while some point of the ruler $K\Lambda$ must be on the point A. Once again, a one-dimensional freedom is left (there are infinitely many points on the line BE that allow the condition). Each choice of point on BE, once again, defines a different angle AEB. Thus the conditions of steps (h) and (i) are parallel. They are also interdependent: AE, $\Gamma\Delta$ being parallel, each choice of point on $B\Delta$ also determines a choice on BE. Of those infinitely many choices, the closer we make Δ to B, the more obtuse angle $\Gamma\Delta E$ becomes, and the further we make Δ from B, the more acute angle $\Gamma\Delta E$ becomes. Thus, by continuity, there is a point where the angle $\Gamma\Delta E$ is right, and this unique point is the one demanded by the conditions of the problem—none of the above being made explicit.

[13] Now—and only now—Δ and E have become specific points.

[14] Note also that the lines AE, $\Delta\Gamma$ are parallel, and also note the right angles at B (all guaranteed by the construction). Through these, the similarity of all triangles can be easily shown (*Elements* 1.29 suffices for the similarity of ABE, $\Gamma B\Delta$. Since Δ, E are right, and so are the sums $B\Gamma\Delta+B\Delta\Gamma$, $BAE+BEA$ (given *Elements* 1.32), the similarity of $\Gamma E\Delta$ with the remaining two triangles is secured as well). *Elements* 6.4 then yields the proportion.

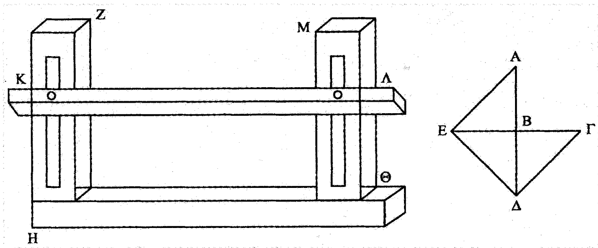

FIGURE 1

III. A SUGGESTED SOLUTION

Let us begin with some preliminary observations. First, something went badly wrong with the ascription to Plato, given the extremely mechanical nature of the solution (it is typical that this solution has many more steps of construction than of argument). This does not appear, then, like a natural flight of fancy on the part of some Neoplatonist—the proof is just not the kind of thing it would be *appropriate* to ascribe to Plato. More likely, then, some scribal error was involved. A simple and anticlimactic solution would be that the proof should have been ascribed to someone else, say a Hippolytus or an Apollonius whose name gradually got mangled in scribal tradition. This is, of course, a possibility: yet a further complication in the evidence prompts us to look further.

This is the tantalizing reference made to Eudoxus. Eutocius makes clear that he has seen another text to which we do not have access. This text had two parts: one, introductory, where a solution was described; the other, containing a solution. The solution alone, or both the introduction and the solution, were ascribed in Eutocius' source to Eudoxus. Seeing that the solution was mathematically incompetent, Eutocius chose to ignore it, throwing away in the process the introduction as well, and leaving us with an enhanced, tantalizing puzzle.

A natural solution at this point would be that Plato's and Eudoxus' names somehow got mixed up, so that an incompetent solution, in some sense by Plato, was ascribed to Eudoxus, whereas a competent, though non-Platonic solution, was ascribed to Plato. This, then, amounts to a new ascription of 'Plato's' solution, this time to Eudoxus; and this was the approach taken by Knorr. The obvious difficulty is that 'Plato's' solution does not make use of any curved lines (a non-use which is difficult to achieve in a solution to this problem: with the single exception of Eratosthenes, all the solutions discussed by Eutocius do make use of some curved lines). Indeed, the importance of curved lines to Eudoxus' historical solution is well attested, in that Eratosthenes referred to his solution as follows:[15]

Eudoxus <solved the problem> with the so-called curved lines.[16]

And [do not attempt] not even that shape which is curved in the lines / That divine Eudoxus constructed.[17]

[15] I assume, following Knorr, the authenticity of Eratosthenes' passage in Eutocius' catalogue of solutions.

[16] Heiberg (n. 1), 90.7–8 (a prose discussion).

[17] Ibid. (n. 1), 96.17–18 (an epigram).

Knorr, typically, offered a brilliant way of squaring this conflict. He has shown how 'Plato's' solution can be derived as a special case of a more general solution—a general solution that does give rise to special curves. (It is indeed an implication of the reference to 'curves' that we have involved here some special curves, and not any regular arcs of circles, say.) By Knorr's reconstruction, then, Eudoxus' original solution, or some later version of it (still ascribed to Eudoxus), had a bipartite structure. First, an abstract approach, giving rise to a curve; second, a concrete application, which we have extant and misascribed to Plato. Somehow, Eutocius' source contained only the second part of this structure, hence Eutocius' puzzlement.

Knorr's discussion is ingenious and possibly true. It is also problematic, in two ways. First, it is necessary to postulate, in this case, a rather complicated course of events. For, in fact, it is very difficult to imagine Eudoxus offering, himself, the concrete solution. This is because Eratosthenes specifically criticizes Eudoxus' solution for its impractical nature, contrasting it adversely with his own solution.[18] Yet in fact 'Plato's' solution is, if anything, easier to implement than Eratosthenes'.[19] Thus one has to assume that 'Plato's' solution was attached to Eudoxus' original solution at some point later than Eratosthenes' time, as a concrete example of how the solution might be obtained in some particular case—both solutions still being ascribed to Eudoxus. This is perfectly possible, but the field of solutions to the problem of two mean proportionals seems to have worked differently. What we see from Eutocius' catalogue—and from Pappus' discussion in Book 3 of the *Collection*—is that solutions to this problem, perhaps more than to any other, were guarded with a strong proprietary urge, authors frequently attaching to themselves solutions distinguished from previous solutions only by some minor differences.[20] Why would anyone offer the relatively important idea of 'Plato's' solution (which, as mentioned above, is striking for its easy implementation), only to hand it out 'for free' to Eudoxus? Thus, while it is perfectly possible that 'Plato's' solution is in some sense inspired by an original discussion by Eudoxus, it is strange to find it first ascribed to Eudoxus, and then mistakenly to Plato. (Finally, a further element of probability against Knorr's interpretation is that it assumes Eudoxus' general solution lying for a hundred years or more before its concrete implementation was even noticed: a possibility, of course, but no more than that.) Knorr's textual course of events is perfectly possible, but somewhat inconvenient, and a smoother reconstruction would be preferable.

But what is more important, Knorr's course of events leaves a real explanatory gap. Knorr does not account for the most puzzling aspect of the puzzle: the lost, incompetent solution. Somehow, this incompetent solution had to get into Eutocius' hands, and it does seem very probable that, had we understood how this solution came to be, we would understand better the circumstances concerning Eudoxus' lost solution, as well as those concerning 'Plato's' solution.

[18] Ibid. (n. 1), 90.8–11: 'As it happens, all of them wrote through demonstrations, and it was impossible to do this practically, by hand (except by the *shortness* of Menaechmus—and this with difficulty).'

[19] Indeed, with Eratosthenes suggesting that the nearest any previous solution gets to being practical is in Menaechmus' solution, some doubt is being thrown on Knorr's reconstruction—after all, even his abstract version is, as one would expect, at least suggestive of the actual concrete method of 'Plato's' solution and thus at least as practical as Menaechmus' extant solution—which is also based on the complicated curved lines of conic sections (Heiberg [n. 1], 78.13–84.7). This line of argument, however, perhaps puts too much confidence in Eratosthenes' testimony.

[20] In Eutocius' catalogue, this is true for the cluster of solutions by Hero, Philo, and Apollonius, and the cluster by Diocles, Pappus, and Sporus.

To begin with, it might be suggested perhaps that Eutocius was wrong to believe the proof was incompetent: perhaps he misread it, confronted with some difficult or corrupt text. This, I believe, would be to underestimate Eutocius, who makes a plausible case for himself as a mathematical philologer—in the case of the famous lost lemma by Archimedes to *SC* 2.4. There, Eutocius rummaged through ancient, corrupt texts, ultimately able to extract a coherent solution from a badly preserved text.[21] It is much more likely, then, to take Eutocius' explicit testimony at face value: he had in front of him a solution to the problem of finding two mean proportionals, which was very patently based on taking a discrete proportion as if it were continuous; furthermore, this solution used no curved lines.

We should also note a further consideration. False solutions are very rare in ancient sources. While Pappus, in Book 3 of the *Collection*, does cite and then criticize at great length, a purported solution to the problem of finding two mean proportionals,[22] this solution is not without some merit, nor is it patently wrong. Most important, it is cited precisely *because* it is criticized. Except in the context of criticism, ancient mathematical compilations tended to be very stringent in their criteria of selection, resulting in a corpus that is nearly mistake-free (the exceptions most often in later scholia, rather than in ancient texts). After all, it was rigour that ancient mathematicians most valued: why they became mathematicians to begin with.

Thus a patently false solution is likely to be transmitted, copied, and recopied only if it has some external, non-mathematical interest. Bearing this in mind, I now offer a possible reconstruction of that solution.

To begin with, let us remind ourselves of the nature of the problem of finding two mean proportionals. Quite simply, it is a geometrical procedure culminating in four lines, being in continuous proportion: the first to the second as the second to the third and as the third to the fourth, or $A{:}B{::}B{:}C{::}C{:}D$. (For the solution to be meaningfully applied, the two extreme lines must be given in advance; this is not a property of the solution I am about to offer—a natural consequence of its falsehood.)

The simplest way one can devise to obtain such four lines, then, is as follows:

Take any two unequal lines: you now have a ratio they define. Cut each of them according to the same ratio, and you have four lines, all in proportion: $(A+B){:}(C+D){::}(A{:}B){::}(C{:}D)$. In a sense, then, we have derived four lines in continuous proportion.

This solution is a sophism, since the four lines are in 'continuous' proportion only in the irrelevant sense that the ratio $(A+B){:}(C+D)$ somehow connects both smaller ratios, $(A{:}B)$ and $(C{:}D)$. In the mathematically significant sense, the proportion is discrete, since the ratio $B{:}C$ is not fixed to be proportional to the other ratios. In fact, it can easily be shown by very elementary considerations, available to Greek mathematicians since at least the early fourth century, that the conditions of the construction define $B = C$. Thus, the construction collapses into three lines in proportion—continuous proportion, indeed—with the caveat, though, that the two extreme lines were not set out in advance.

I suggest, then, that we have before us a patently false solution, which does not use curved lines, and which could have been transmitted for the non-mathematical interest of its being ascribed—not without reason—to Plato. Two questions remain: first, in what manner could this solution have been ascribed to Plato? Second, how did

[21] Heiberg (n. 1), 130.17–132.18. For further discussion, see Netz (n. 4).

[22] See the discussion in S. Cuomo, *Pappus of Alexandria and the Mathematics of Late Antiquity* (Cambridge, 2000).

Eutocius' source lead him to ascribe, on the one hand, this false solution to Eudoxus and, on the other hand, another, totally unrelated, solution to Plato?

The 'solution' reconstructed above is related to Plato in an obvious way. In the famous Divided Line passage (*Rep.* 509d6–8) Plato defines four lines precisely by the construction above. It is, in fact, one of the few passages in Plato where a description of a mathematical operation is made perfectly determinate, so that there is no scholarly debate concerning the interpretation of the mathematics as such.[23]

There are of course many puzzles concerning this passage, involving the nature of its metaphorical use in elucidating Plato's ontological and epistemological ladders. A specific puzzle, arising from the mathematical features of this construction, is the equality of the second and third sections (metaphorically, the visible realm, and the realm of mathematics). Here is not the place to discuss any of those puzzles, but a word ought to be said for its relevance to the problem of finding two mean proportionals. For the suggestion of this article is very simple: some author, probably in late antiquity, compiling a list of solutions to the problem of finding two mean proportionals, added in a paraphrase of the Divided Line passage, through either mathematical incompetence (the late author did not realize this was a false solution), or philosophical insight (the late author realized that the solution, while false, has a certain intended resemblance to the problem of finding two mean proportionals, and is worthy of being put in a collection of such solutions). While this is not necessary for my argument, I shall now suggest that the late author I envisage here had in fact understood, perhaps inadvertently, a deep fact about the *Republic*.

In fact, this problem of two mean proportionals is prominently flagged by Plato in the *Republic*, in another curious passage, worth considering in some detail. In the Curriculum passage—still thematically related to the Divided Line passage—Socrates surveys the mathematical sciences appropriate for the guardians' education. From geometry, they pass to astronomy, and then Socrates retrieves his steps (*Rep.* 528a6–b5):

> [Socrates:] Just now, we have incorrectly taken that which follows geometry. . . . After the plane [the subject matter of geometry] we have now taken the solid as being in motion [the subject matter of astronomy]. But the correct thing is, following the second growth/dimension, to take the third. And this is, say, about the growth/dimension of cubes, and about that which has depth. [i.e., stereometry]—So it is, said [Glaucon]. But then, those things do not seem to have been studied at all!

Here follows a long exchange building on Glaucon's last remark: a hypertrophy of the irony of hindsight, often seen elsewhere in Platonic dialogues. The characters, Socrates and Glaucon, are made to speak about a discipline, which, they stress, does not exist, as if characters in a story set in Bern, 1900, would discuss the desirability of a certain theory of relativity. Why Plato should have all this is of course an open question, but the implication, of the outlined historical development, is persuasive: it is likely that, in the fifth century, there was little we could recognize as 'stereometry', much more of it in the fourth century. In particular, while the problem of two mean proportionals was merely associated in the fifth century, by Hippocrates of Chios, with stereometry, it was only in the fourth century that this problem was first solved by

[23] See G. E. R. Lloyd, 'The Meno and the mysteries of mathematics', *Phronesis* 37 (1992), 166–83, for an account of Plato's, more typical, intended indeterminacy in mathematical descriptions.

Archytas and Eudoxus.[24] The fact is, there is a very deep connection between the three-dimensional and the two mean proportionals. Take a cube, and make it grow by a certain ratio, say that of two to one; to do so, you need to make the side of the cube (the only object to which you have geometrical access) grow by another ratio. Doubling the side of the cube, after all, would result in a cube eight times, not twice, greater. To obtain a cube twice as great as the original one—to 'duplicate the cube', as they say—you must make the side grow by what we would call the cubic root of two. The Greeks would say that you must find the second term in a sequence of four lines in continuous proportion: so, to make cubes grow by definite ratios is the same as to find two mean proportionals. So much was discovered by Hippocrates of Chios: and this is what 'stereometry' would, above all, be for the ancient Greeks.

Now, when we think about stereometry, we may think perhaps more often of the three-dimensional equivalents of Euclid's plane geometry (which are indeed developed by Euclid in his Book 11 of the *Elements*): how certain planes are parallel to another, when a line is in right angles to a plane, and so on. As is often the case with the *Elements*, the results developed there are of great use, but of little intrinsic interest. The modern tradition of geometry was often interested in foundational questions, so that what for Euclid and his audience was a tool becomes for us a fascinating subject in its own right. There is no reason to think the Greeks were as fascinated by Book 11 of the *Elements*. We know, after all, what fascinated them in mathematics: ratio and proportion. Immediately following upon Book 11, Euclid moves on to the more exciting results of Book 12 and Book 13: in the first, a simple ratio (1:3) is found between complex figures (pyramids and prisms, cones and cylinders); in the second, much more complex ratios are found between regular solids and the sphere in which they are inscribed. It is indeed remarkable how Euclid focuses, for the solid figures, on this one aspect of ratio—missing the point, for us, concerning the topological constraints on faces, edges, and vertices. But of course this insistence on ratio fits the general pattern in Greek mathematics, where ratio and proportion are the dominating theme. Thus the study of three-dimensional figures is primarily the study of ratios between such figures, that is, more than anything else, the study of the problem of two mean proportionals. Plato, in fact, says this as clearly as he can, given the constraints of the irony of hindsight: 'And this is, say, about the growth/dimension of cubes, and about that which has depth.' Given the above description, it is tempting to read this as if Socrates refers, first, to the primary problem of stereometry—the growth of cubes, that is the finding of two mean proportionals—and then to a more abstract characterization of the field: the study of objects possessing depth. The word *αὐξή*, which I translate with an indeterminate growth/dimension, neatly captures the duality: strictly speaking, it just means 'growth'. It is true that, in context, it also means something like 'dimension', but this is precisely because ratios are so central to the Greek understanding of geometrical objects: solid objects, to the Greek, have three 'growths'—three dimensions to their ratios.[25]

All of this, then, creates a probability that Plato, in writing the *Republic*, was aware of the connection between stereometry and the problem of finding two mean

[24] So much we learn from Eratosthenes' testimony (Heiberg [n. 1], 88.17–23, 90.4–8. Archytas' solution—a three-dimensional marvel—is also preserved (ibid. 84.12–88.2).

[25] Finally, it is relevant to note that Plato very explicitly connects the nature of solid bodies with the mathematical structure of two mean proportionals, in *Tim.* 32a7–b3: famously, the four-term structure yielded by the problem of two mean proportionals is taken by Plato to explain the need for four elements out of which solid bodies are made.

proportionals, as well as of the historical development of both. In other words, he made a point of stressing that, to the speakers of the *Republic*, a solution to the problem of finding two mean proportionals was not yet available. It is thus appropriate for him to offer a passage where the speakers contemplate what appears, at first glance, very much like a solution to this problem, yet is actually false. The Divided Line passage, under this interpretation, is permeated by the irony of hindsight, emphasizing the limits of the mathematics available to the historical Socrates, ironically self-referential in its—probably intentional—collapse of four segments into three. Bringing in mathematics, Socrates fumbles his mathematics, in a specific, meaningful way, assuming complex ratios can be made easy. What the philosophical significance of this may be, I shall not discuss here.

The first part of my suggestion, at any rate, has at least some plausibility: a late ancient author could have compiled a list of several solutions to the problem of finding two mean proportionals, putting in a mathematized version of the Divided Line passage, describing this as 'Plato's solution'. This certainly would have resulted in a patently false solution, using a discrete proportion as if it were continuous, but not using any curved lines. The question remains, why would Eutocius think this solution was by Eudoxus? And why would he ascribe another solution still to Plato?

The most natural way to answer this is to assume some defect in Eutocius' manuscript source. There are myriad stories that can be told, but there is one I can think of that is extremely simple. Imagine the archetype to Eutocius' source contained, in this order:

An introduction and solution, ascribed to Eudoxus.
A solution, ascribed to Plato.
A solution, ascribed to an author X whose identity escapes us.

(To clarify my suggestion I present the ascriptions as marginal comments, but nothing relies on the specific layout.) Then the physical structure would be something like this:

'As Eudoxus'	Introduction by Eudoxus	Solution by Eudoxus
'As Plato'	Solution by Plato (= mathematized Divided Line)	
'As X'	Solution by X (= 'Plato's' solution)	

Now imagine that, in the source immediately used by Eutocius, the solution by Eudoxus got somehow dropped – a major oversight, but not at all an impossibility. Then the structure of the main text would become:

Introduction by Eudoxus
Solution by Plato (= mathematized Divided Line)
Solution by X (= 'Plato's' solution)

And now, introducing the marginal comments, there are only two solutions to affix them to—a hole missed when buttoning the shirt, and you are left with a uselessly dangling button. Thus the final, third marginal comment is simply ignored, since there is nowhere to affix it:

'As Eudoxus'	Introduction by Eudoxus
	Solution by Plato (= mathematized Divided Line)
'As Plato'	Solution by X (= 'Plato's' solution)

This, of course, precisely fits the situation as described by Eutocius.

It remains to note that it is likely that, following 'Plato's' solution, Eutocius does move to a different source or sources: the following clusters of solutions—Hero–Philo–Apollonius and Diocles–Pappus–Sporus—are organized by their geometrical similarity, are often explicitly referred to works that Eutocius was likely to know at first hand, and generally speaking do not have the exotic nature of the solutions mentioned at Eutocius' introduction and at the 'as Plato' passage. It thus seems reasonable to assume that, at first, Eutocius worked from a source of a historical and literary, no less than mathematical, interest, listing three very early solutions. Indeed, we often hear about a literary exposition of the problem of finding two mean proportionals, connecting it specifically to Plato: according to this literary enactment, the Delians received an oracle demanding that they double a certain cubical altar. Approaching Plato, seeking a solution, they learned instead the true meaning of the oracle: it signified, as oracles do, indirectly, an exhortation—the Greeks should study geometry![26] It seems that this story was first offered in a lost and little-known work by Eratosthenes, the *Platonicus*. Theon of Smyrna's work—where we learn about the origin of the Delian story in the *Platonicus*—perhaps reflects the ancient context in which the *Platonicus* and similar works were produced. Here were popularizations of mathematics, so to speak, reflecting upon themes in Platonic writings. Either in direct response to the *Platonicus*, or simply in a context similar to that of the *Platonicus*, an ancient author could easily have produced a sequence of solutions to the problem of two mean proportionals. The solutions could be chosen by their literary-historical credentials, and a mathematization of the Divided Line could well be thrown in, perhaps tongue in cheek.[27]

Speculation may perhaps have gone too far, and now is the time to stop. I believe this article offers a possible interpretation of the puzzle in the passage from Eutocius. To the extent that this interpretation may be true, it would provide an interesting example of what may be called 'Plato's Mathematical Construction'. As late readers, often somewhat Neoplatonist, came to read mathematics, and as the same readers, now with a certain mathematical education, came to read Plato, a new Platonic-mathematical complex came to be. Seen through this perspective, the early history of mathematics came to be seen as leading to, and from, Plato; while Platonic writings were seen to contain specific mathematical statements, to be unpacked by appropriately trained readers.[28] Plato, then, was construed as a mathematician.

[26] The story appears twice in Plutarch (*Moralia* 386e, 579a–d), once in Theon of Smyrna (Hiller 2.3–12), and, finally and most important, in the Eratosthenes fragment preserved by Eutocius (Heiberg [n. 1], 88.23–90.4). Theon explicitly cites his story from Eratosthenes' *Platonicus*. Plutarch's version in 579a–d is by far the longest version and, with its dialogue setting (*The Sign of Socrates*), might perhaps reflect closely a similar source in Eratosthenes; but this is mere speculation, which perhaps does not do justice to Plutarch's originality.

[27] Plutarch's *Moralia* 579a–d is suggestive: the Delians are seeking Plato 'as a geometer' (579a–b); Plato then explicates the deep meaning of the oracle adding that, more practically, they should ask Eudoxus or Helicon (579c). Could a source of this kind prompt an author to compile a sequence of three solutions—Eudoxus', Plato's, Helicon's? A sheer speculation, of course, but useful in delineating the kind of historical process leading to our passage in Eutocius.

[28] L. Zhmud, 'Plato as "architect of science"', *Phronesis* 43 (1998), 211–44, surveys the general

If I am right, finally, Eutocius' position in this history becomes doubly ironic. Himself trained in this philosophical-mathematical tradition,[29] Eutocius clearly was stronger in his mathematics than in his philosophy: as it were, a failed Neoplatonist. So much so, that he fell for the Divided Line solution, failing to see the joke; though to his credit it must be said that, with his source being corrupt, it is easy to understand his mistake.

Finally, an apology for my own article. The argument offered here is tentative, in the sense that one cannot tell how much the story offered here is likely to be true. But a truth has been shown, I believe, through the chain of argument itself. We have followed possibilities and plausibilities: they need not be true, but the fact of their being plausible or possible is a reality well worth learning. We have followed the shadows of ancient events to whose reality we may never have access, learning, in the process, something real about Platonic mathematical philosophy and about its ancient transmissions and transformations.

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phenomenon of this mathematical construction of Plato. Perhaps the example closest to the process suggested in this article is Simplicius' account of Plato's role in the early history of astronomy, perhaps derived from Sosigenes. According to this account (Simplicius *in De Caelo* 488.20–4), Plato set an explicit task to mathematical astronomers, 'to find what are the uniform and ordered movements by the assumption of which the apparent movements of the planets can be accounted for'. This is clearly a reading—as if it was meant to be read in such theoretically precise terms—of a vague and discursive passage in the *Laws*, 821b5–822c9. Astronomy was extracted out of Plato; so, I suggest, was stereometry.

[29] The commentaries to SC are dedicated to Ammonius, apparently the same as the teacher of Simplicius. Eutocius and Simplicius, indeed, could have been classmates; yet Eutocius' commentaries hardly betray any philosophical training.